

$$\max x_1 - 2x_2 + 3x_4$$

$$\text{s.t. } x_2 - 6x_3 + x_4 \leq 4$$

$$-x_1 + 3x_2 - 3x_3 = 0$$

$$6x_1 - 2x_2 + 2x_3 - 4x_4 \geq 5$$

$$x_2 \leq 0$$

$$x_4 \geq 0$$

$$x_1, x_3 \in \mathbb{R}$$

(1)
(2)
(3)

Constructing a dual program: Using the recipe

Part 1.

Let us write vectors b and c from right hand side of the constraints and objective function respectively

$$c = \begin{pmatrix} 1 \\ -2 \\ 0 \\ 3 \end{pmatrix}, \quad b = \begin{pmatrix} 4 \\ 0 \\ 5 \end{pmatrix}$$

Dualization Recipe

	Primal linear program	Dual linear program
Variables	x_1, x_2, \dots, x_n	y_1, y_2, \dots, y_m
Matrix	A	A^T
Right-hand side	\mathbf{b}	\mathbf{c}
Objective function	$\max \mathbf{c}^T \mathbf{x}$	$\min \mathbf{b}^T \mathbf{y}$
Constraints	i th constraint has \leq \geq $=$ $x_j \geq 0$ $x_j \leq 0$ $x_j \in \mathbb{R}$	$y_i \geq 0$ $y_i \leq 0$ $y_i \in \mathbb{R}$ j th constraint has \geq \leq $=$

Part 2

Primal program has 3 constraints.
 Therefore the dual program
 has 3 variables as follows:
 (according to the recipe)

$$\begin{array}{l}
 y_1 \geq 0 \\
 y_2 \in \mathbb{R} \\
 y_3 \leq 0
 \end{array}
 \begin{array}{l}
 \leftarrow \\
 \vdots \\
 \leftarrow
 \end{array}
 \begin{array}{l}
 \text{(corresponds to} \\
 \text{the 1st} \\
 \text{constraint} \\
 \text{in} \\
 \text{primal)}
 \end{array}$$

Part 3.

From the variables of primal (x_1, x_2, x_3, x_4) we can get the correct inequalities of the dual:

Thus, the first constraint is "="

the second "≤"

the third "="

the fourth "≥"

Part 4.

Matrix of constraint A can be written:

$$A = \begin{pmatrix} 0 & 1 & -6 & 1 \\ -1 & 3 & -3 & 0 \\ 6 & -2 & 2 & -4 \end{pmatrix}$$

Final Part

Take all parts and combine them together:

$$\min b^T y = 4y_1 + 5y_3$$

with constraints given by

$$A^T y = \begin{array}{r} -y_2 + 6y_3 = 1 \\ y_1 + 3y_2 - 2y_3 \leq -2 \\ -6y_1 - 3y_2 + 2y_3 = 0 \\ y_1 - 4y_3 \geq 3 \end{array}$$

where $c = \begin{pmatrix} 1 \\ -2 \\ 0 \\ 3 \end{pmatrix}$

and variables satisfying

$$y_1 \geq 0$$

$$y_2 \in \mathbb{R}$$

$$y_3 \leq 0$$